2. Scaling the heights of the atmosphere

or the problem of getting up and staying up

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References


Understanding the Earth’s atmosphere

- Ionosphere (Aurora)
- Mesosphere
- Ozone Layer
- Stratosphere
- Tropopause
- Troposphere
- Earth
Question: Are the boundaries for the atmospheric regions the same everywhere on the Earth?

No. The Earth is not a perfect sphere.

The World Geodetic System (WGS) models the Earth as an oblate spheroid

\[
\begin{align*}
\text{equatorial axis} & \quad = \quad 6,378,137.000 \text{ m} \\
\text{polar axis} & \quad = \quad 6,356,752.314 \text{ m} \\
\text{polar tropopause} & \quad = \quad 6 \text{ km} \\
\text{equatorial tropopause} & \quad = \quad 17 \text{ km}
\end{align*}
\]
**Question**: Is the Earth’s atmosphere uniform?

<table>
<thead>
<tr>
<th></th>
<th>0 km</th>
<th>20 km</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Air pressure (N/m²)</strong></td>
<td>101 325</td>
<td></td>
</tr>
<tr>
<td><strong>Air density (kg/m³)</strong></td>
<td>1.225</td>
<td></td>
</tr>
<tr>
<td><strong>Air temperature (°C)</strong></td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

**Question**: Any implications for flight vehicles?
**Question**: So how are the boundaries defined? By pressure, density or temperature?

Air temperature falls at a constant rate in the troposphere.

From the tropopause, the temperature remains constant at -60 °C until ≈20 km above S.L.

The lower stratosphere is the limit for atmospheric flight.
How high can artillery go?
What can we see at high altitudes?
More conventional example

- RQ-4 Global Hawk High Altitude, Long Endurance (HALE) UAV
- Mission: Fly 1,200 miles and remain on site for 24 hrs at 18km altitude
- Sensor suite: Electro-optical, IR, synthetic aperture radar, ground moving target indicator.
- Capability: scan 40,000 nautical square miles (63,000 km²) in 24 hrs
Why fly at high altitudes?

• Jet stream: fast moving current of air at altitudes of levels of 10-15 km caused by temperature differences.

• Typically $O(10^3)$ km long, $O(10^2)$ km wide, and a few km thick.

• Wind speeds from 55 km/h to 120 km/h causing air turbulence.
Aerodynamic forces on a flight vehicle scale as:

\[ \text{Aerodynamic force} \propto \rho V^2 \]

Note the dependence on \( V^2 \)
For missiles, there are two important aerodynamic forces

\[
\text{Axial force } A = \frac{1}{2} \rho V^2 S C_A \\
\text{Normal force } N = \frac{1}{2} \rho V^2 S C_N
\]

These forces are aligned with the missile body and not the velocity

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The symbols are:

\[ S \] : reference area (m\(^2\)) e.g. missile cross section area

\[ C_A \] : axial force coefficient (non dimensional)

\[ C_N \] : normal force coefficient (non dimensional)

\[ \frac{1}{2} \rho V^2 \] : dynamic pressure (N/m\(^2\))
Equivalently we can represent the aerodynamics forces as lift and drag forces aligned with the velocity

<table>
<thead>
<tr>
<th>Force</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lift force L</td>
<td>$L = \frac{1}{2} \rho V^2 S C_L$</td>
</tr>
<tr>
<td>Drag force D</td>
<td>$D = \frac{1}{2} \rho V^2 S C_D$</td>
</tr>
</tbody>
</table>

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Ex: Estimate $C_L$ for the AGM 65

Flight conditions

mass : 300 kg
speed : 320 m/s
altitude: sea level
diameter: 0.3048 m
For level flight,

\[ S = C_L \]
**Ex**: Speed/altitude variation for vertical launch AMM
Question: Estimate the terminal velocity to engage a target at sea level
At 10 km,

At S.L 0 km,
Aerodynamic flow parameters

Missile/projectile airspeeds can range from $10^0 – 10^3$ m/s

For this range of speeds, airflow characteristics are determined by 2 important parameters:

1. Reynolds number $Re$

2. Mach number $M$
Reynolds number

1. Air is “sticky” or viscous
2. From the missile’s viewpoint, the air at the surface is stationary

$V = \text{missile airspeed}$

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3. The thin region where the air flow builds up its speed is called the boundary layer.

4. The Reynolds number is a measure of the importance of this viscous effect.

\[ Re = \frac{(\rho V^2)}{(\mu V / L)} = \frac{VL}{\nu} \]

- \( \rho \): mass density
- \( V \): velocity
- \( L \): reference length
- \( \mu \): coefficient of viscosity
- \( \nu = \mu/\rho \): kinematic viscosity
Ex: What are typical missile Reynolds numbers?

Using the AGM-65 at S.L.

\[ V : 320 \text{ m/s} \quad L : 0.3048 \text{ m (diameter)} \]
\[ \nu : 1.4607 \times 10^{-5} \text{ m}^2/\text{s} \quad \text{(kinematic viscosity for air at S.L.)} \]

\[ Re = \frac{V \cdot L}{\nu} \]
The Mach number

1. Air is compressible.

2. A moving missile disturbs the surrounding air.

3. These disturbances e.g. pressure variations, take a finite time to propagate at the speed of sound through the surrounding air.

4. The Mach number measures the importance of this compressibility effect.

\[
M = \frac{\text{airspeed}}{\text{(speed of sound)}} = \frac{V}{a}
\]

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Example: Disturbance propagation $M < 1$

Consider the distances travelled by the disturbance and the missile in 1s

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Example: Disturbance propagation $M > 1$

Consider the distances travelled by the disturbance and the missile in 1s.
So for $M > 1$, there is a discontinuity in the flow field “seen” by the missile.

Air properties like pressure, temperature and density changes sharply across the discontinuity or *shock*.

Schlieren photo of shock waves

Light is refracted differently because of changes in air density.

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The shape of the shock wave depends on the shape of the object. Shocks created by high speed flight can be annoying ....
Effects of a shock (sonic boom)

On the ground

On humans

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Condensation due to sudden changes in air temperature and pressure
Classification of flow regimes via speed

- $M < 0.8$  subsonic  incompressible aerodynamics
- $0.8 < M < 1.2$  transonic  localized compressibility effects
- $1.2 < M < 5$  supersonic  compressible aerodynamics
- $M > 5$  hypersonic  aerodynamic heating