4. Energy Principles - Preliminaries
4.1 Displacement Coordinates

A quantity used to specify the configuration of a system

The “displacement “is used in a generalized sense.

It may be a change in distance (Cartesian coords) or a change in angular position (polar coords)
4.2 Virtual Displacement

An imaginary, infinitesimal change in the configuration of a system consistent with constraints

Example: Bar of length L pivoted at O.

The bar rotates by a virtual displacement of $\delta\theta$.

What is resulting virtual displacement of the mass at A?
The relation between the coordinates is

Using the chain rule of differentiation, the virtual displacements are :

Question : Are these virtual displacement consistent with the constraint \( x^2 + y^2 = L^2 \) ?

Answer : Yes (why ? )

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4.3 Generalized Coordinates

A set of *independent* displacement coords which are *consistent* with the constraints and are *sufficient* to describe the configuration of the system.

**Question**: Which set of displacement coords qualify as generalized coords?

- a) \{x, y\}
- b) \{r, \theta\}
- c) \{\theta\}

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4.4 Virtual Work and Generalized Forces

The virtual work $\delta W$ is the work of the forces acting on the system as it undergoes a virtual displacement.

If we write the virtual work as:

$$\delta W = \sum_{i=1}^{N} Q_i \delta q_i$$

The term $Q_i$ is referred to as a *generalized force* (Why?)

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4.5 The Principle of Virtual Displacements

For any arbitrary virtual displacement, the sum of the virtual work done by the applied forces and inertia forces is zero

\[ \delta W_{\text{applied}} + \delta W_{\text{inertia}} = 0 \]

Note: This provides another way to derive equations of motion
Example : Driven Pendulum

1. Pendulum of mass $m$ and length $L$

2. In addition to gravity a force is applied perpendicular to string

$$F_{\text{applied}} = F \{-\sin \theta, \cos \theta\}$$

3. Derive the equations of motion using the principle of virtual displacements
Step 1: Select the coordinates as:

Step 2: Suppose the pendulum undergoes a virtual rotation of $\delta \theta$

Step 3: Find the virtual work done by the applied forces is:

$\delta W_{\text{applied}} = \ldots$

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Step 4: Define the inertia forces (note the -ve sign)

\[ F_{\text{inertia}} = \]  

Step 5: Find the virtual work due to inertia forces

\[ \delta W_{\text{inertia}} = \]
Step 6: Invoke the principle of virtual displacements

\[ \delta W_{\text{applied}} + \delta W_{\text{inertia}} = 0 \]

Since this must hold for any \( \delta \theta \),

Hence the equation of motion is:

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4.6 Hamilton’s Principle

Hamilton’s Principle states that the configuration of a system between two instances in time changes such that:

\[ \int_{t_1}^{t_2} \delta(T - V) dt + \int_{t_1}^{t_2} \delta W_{nc} dt = 0 \]

- \( T \) : kinetic energy of system
- \( V \) : potential energy of system
- \( W_{nc} \) : virtual work done by non conservative forces
- \( \delta() \) : virtual change of quantity in brackets