Problem Definition

What happens to the wing’s natural frequency if the missile and launcher is removed?

wing half span = 4.57 m

mass of empty aircraft = 8 400 kg
mass of aircraft w/2 missiles = 12 138 kg
mass of missile = 85.5 kg
mass of launcher = 27.0 kg

missile length = 2.87 m

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1. Wing spars as an equivalent cantilevered beam
2. Missile + launcher as a lumped mass
Applying the Energy Method

*For a beam*, the elastic strain energy and kinetic energy are given by:

\[
V = \frac{1}{2} \int_{0}^{L} EI \left( \frac{\partial^2 v}{\partial x^2} \right)^2 dx
\]

\[
T = \frac{1}{2} \int_{0}^{L} \rho A \left( \frac{\partial v}{\partial t} \right)^2 dx
\]

**Question**: What about the mass?
Using $N$ assumed modes, we let $v(x,t) = \sum_{i=1}^{N} \psi_i(x) q_i(t)$

The elastic strain energy for the beam is:

$$V = \frac{1}{2} \int_0^L EI (\psi_1''(x) q_1(t)) + ... + \psi_N''(x) q_N(t))^2 \, dx$$

$$= \frac{1}{2} q^T [K]_{N \times N} q$$

where $q^T = \{q_1, ..., q_N\}$

$$K_{ij} = \int_0^L EI \psi_i''(x) \psi_j''(x) \, dx$$
The kinetic energy (with the mass included) is:

\[ T = \frac{1}{2} \int_0^L \rho A (\psi_1(x) q_1'(t)) + \ldots + \psi_N(x) q_N'(t))^2 \, dx \]

\[ + \frac{1}{2} m (\psi_1(L) q_1'(t)) + \ldots + \psi_N(L) q_N'(t))^2 \]

\[ = \frac{1}{2} q'^T [M]_{N \times N} q' \]

where \( q'^T = \{q_1', \ldots, q_N'\} \)

\[ M_{ij} = \int_0^L \rho A \psi_i(x) \psi_j(x) \, dx + m \psi_i(L) \psi_j(L) \]

What does this imply?

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Therefore by Lagrange’s formulation, the N DOF EOMs for the free vibration:

\[ M \ddot{q} + K q = 0 \]

Question: So what's the effect of the mass on the first natural frequency?
1 DOF Approximation

Using only 1 mode \( v(x,t) = \psi_1(x) q_1(t) \)

\[
K_{11} = \int_0^L EI \psi_1''(x) \psi_1''(x) \, dx
\]

\[
M_{11} = \int_0^L \rho A \psi_1(x) \psi_1(x) \, dx + m \psi_1(L) \psi_1(L)
\]

Now select an assumed mode ... 

What’s your conclusion?

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