Lecture 3 : Static Longitudinal Stability

Or the balancing of lift forces and pitching moments

1.0 Trim condition

An aircraft is *trimmed* if there are no nett forces and moments acting on it

E.g. The nett pitching moment coefficient is zero

$$M \quad = \quad 1/2\, \rho\, V^2\, S\, c\, C_m \quad = \quad 0$$

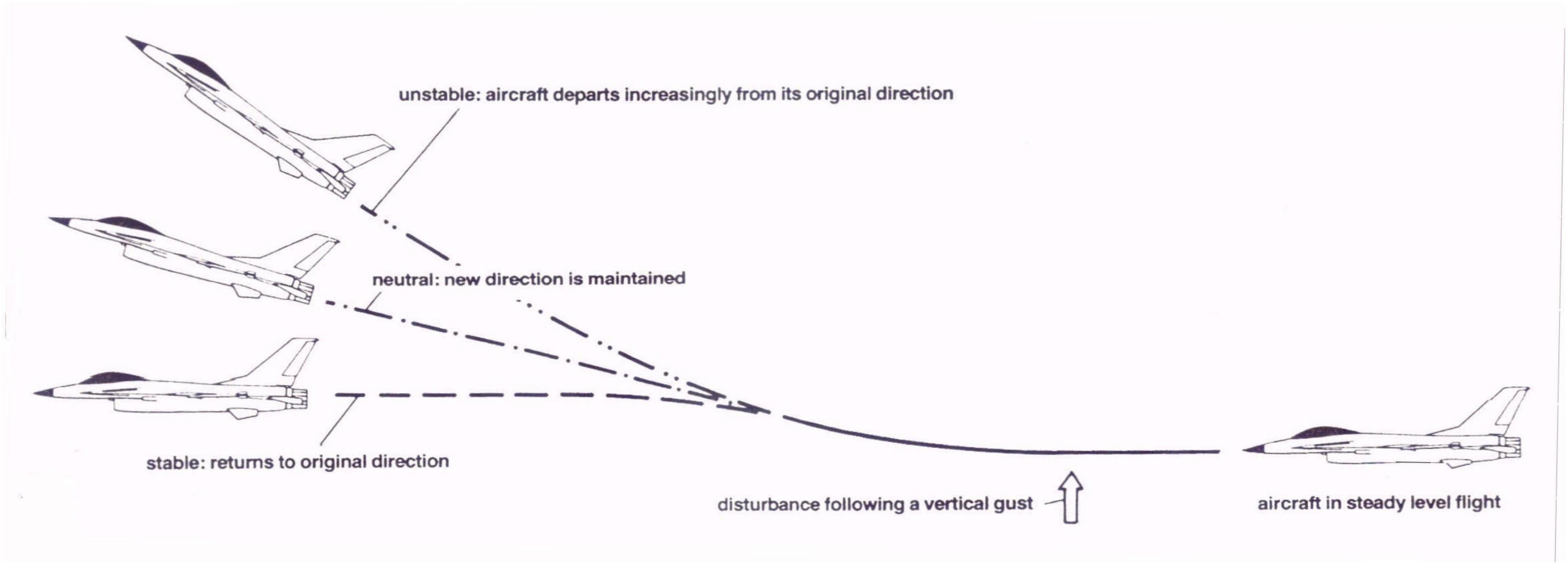
pitching moment coefficient

1.1 Concept of static stability

Static stability refers to the *tendency* of the aircraft to develop forces or moments to return to its trim condition when disturbed

Question : What does that mean ?

Figure 1.1 : Static stability



1.2 Longitudinal static stability

Longitudinal static stability refers to the tendency of the aircraft to return to its trim condition after a nose up or nose down disturbance

This implies that C_m must vary with AOA in a certain manner !!

Question : What manner ?

Figure 1.2 : Cm variation with AOA

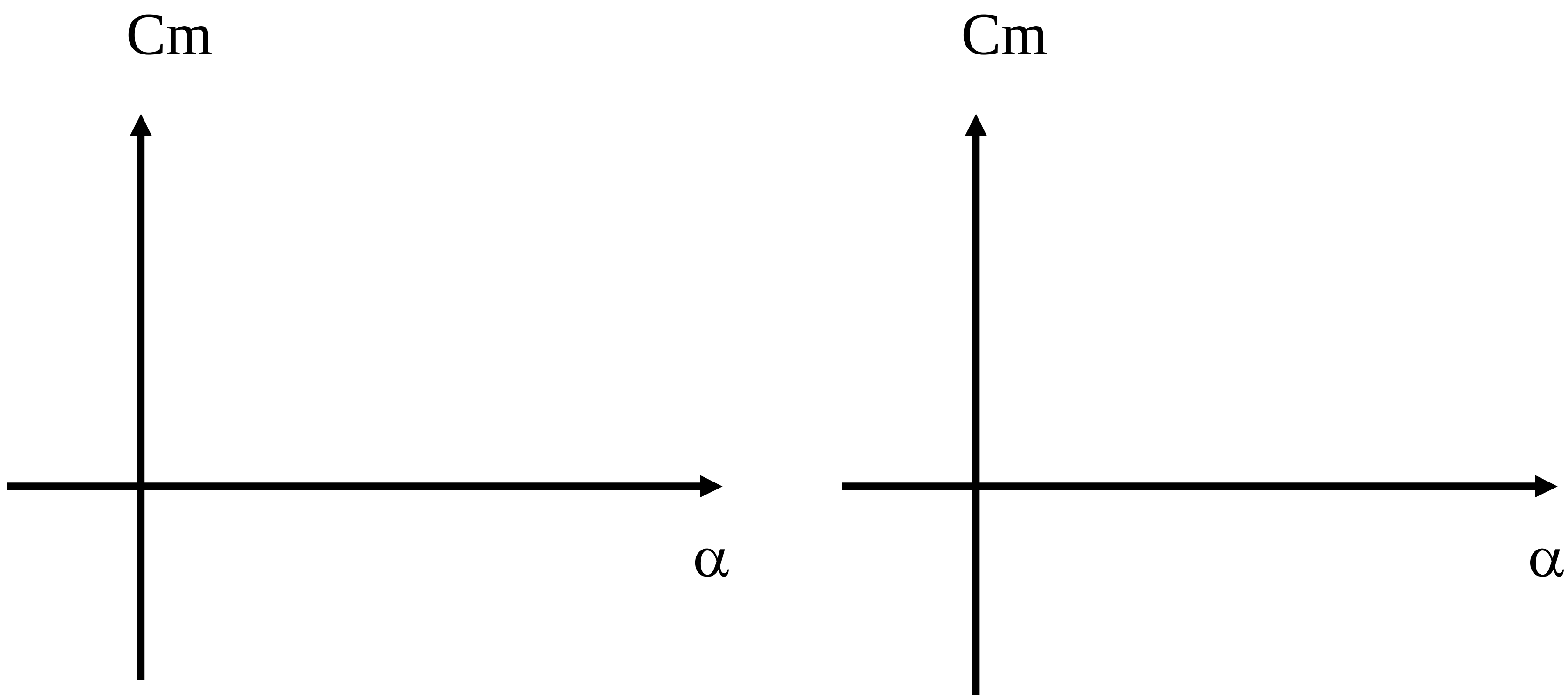
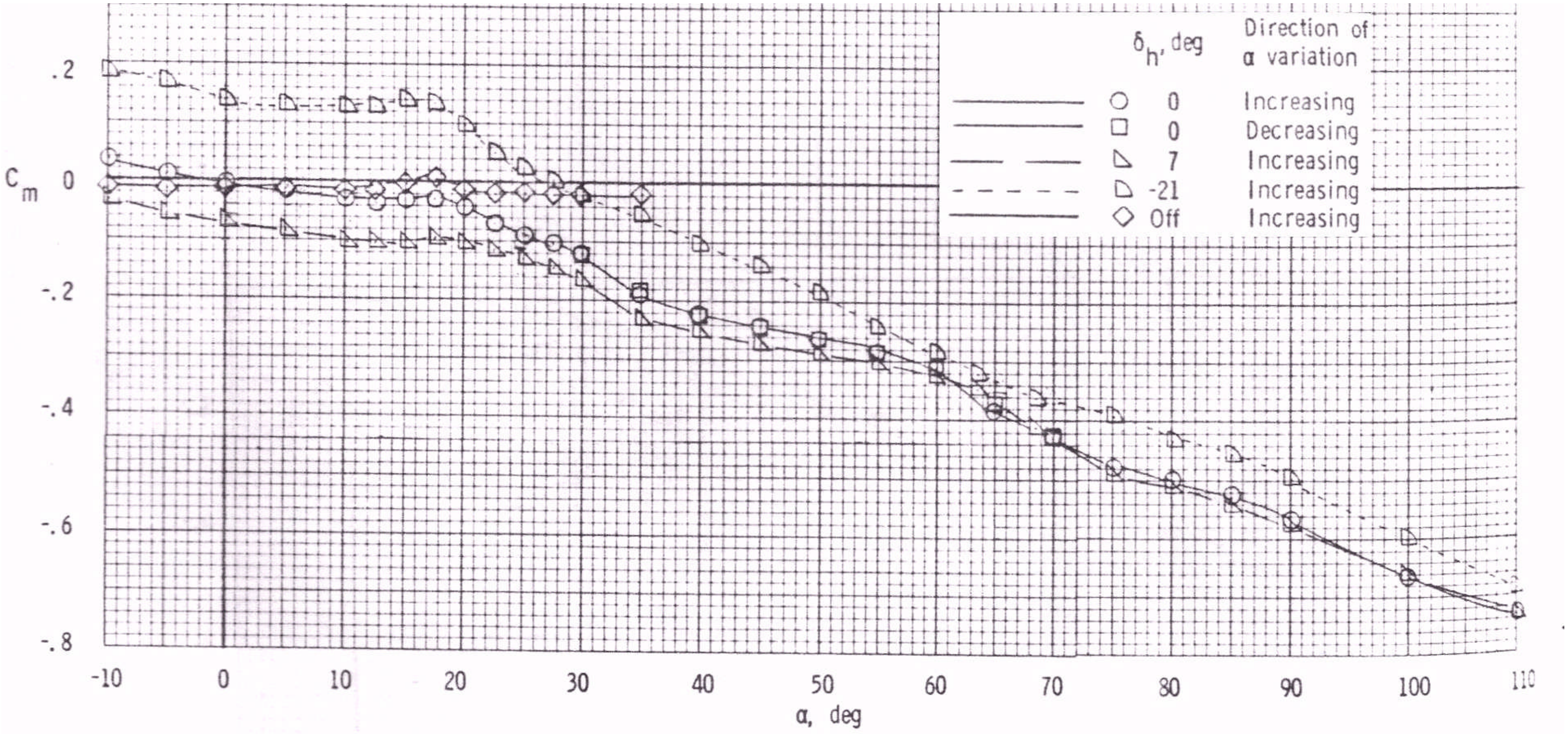


Figure 1.3 : F4 wind tunnel data - pitching moment



Source : NASA Technical Note D 6425

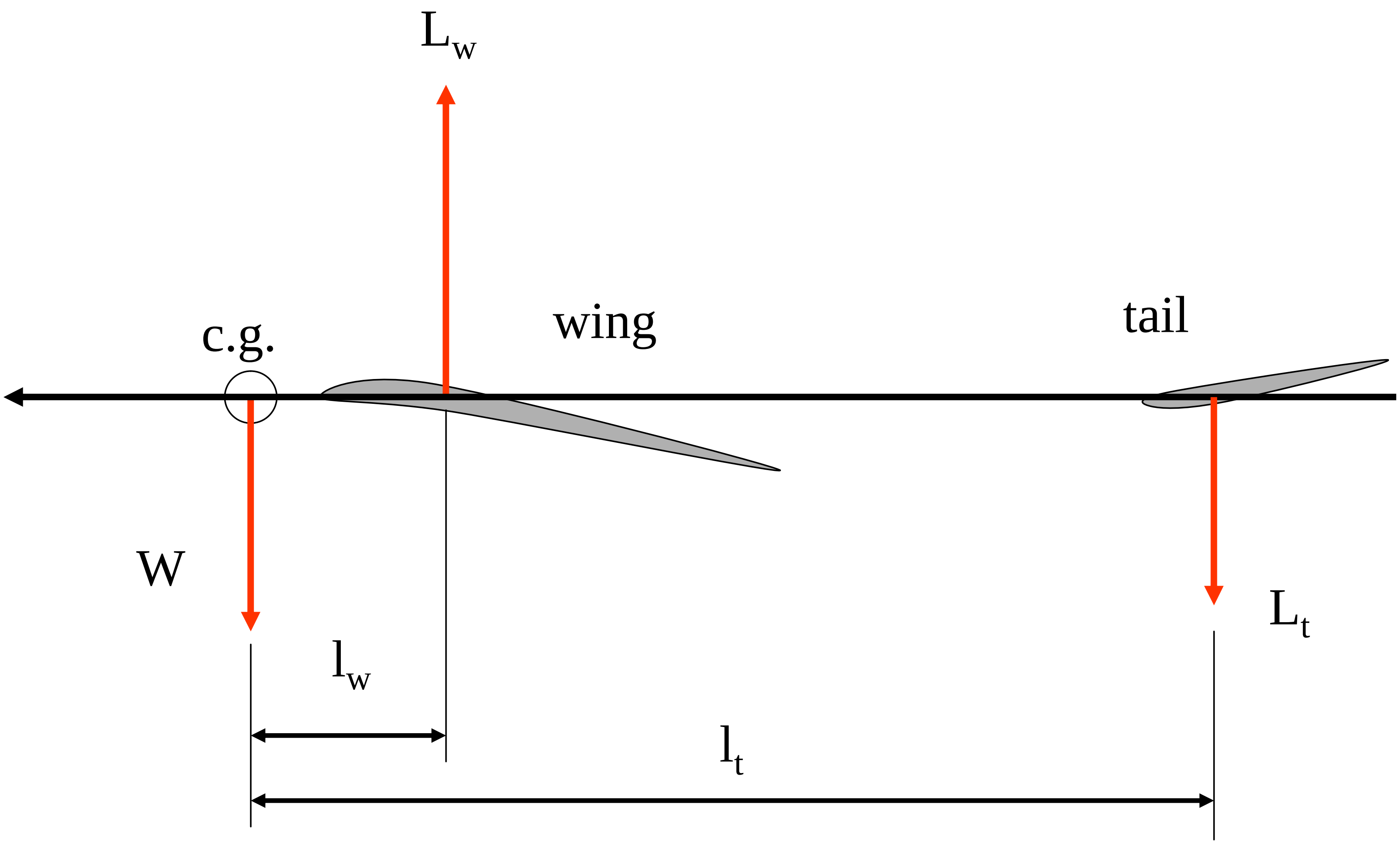
Moral

A statically stable aircraft must have $\partial Cm / \partial \alpha < 0$

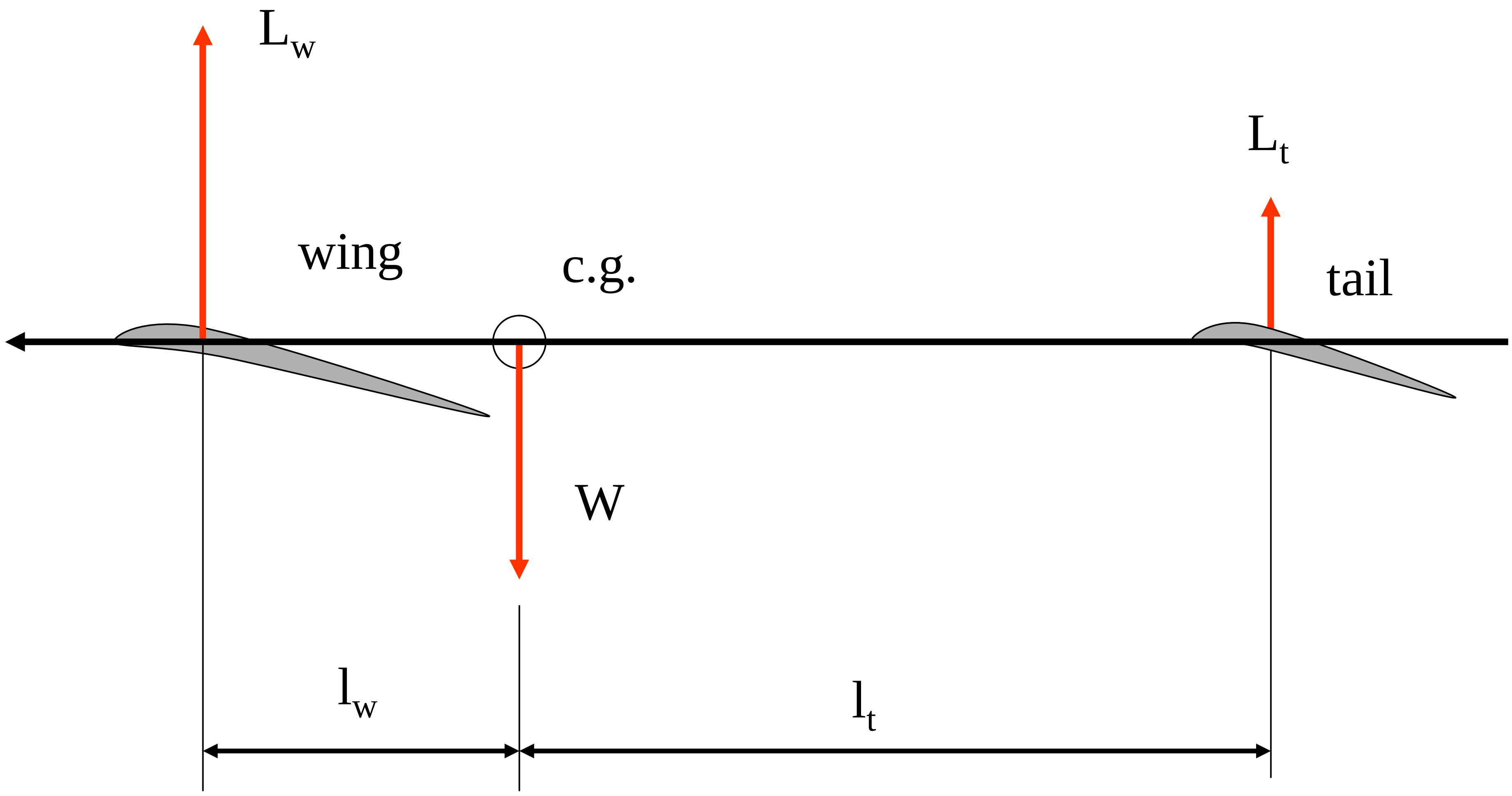
Note : Aeronautical engineers usually write $\partial Cm / \partial \alpha$ as Cm_{α}

Question : Is static stability always a good thing ?

1.3 Tail configuration - statically stable



1.4 Tail configuration - statically unstable



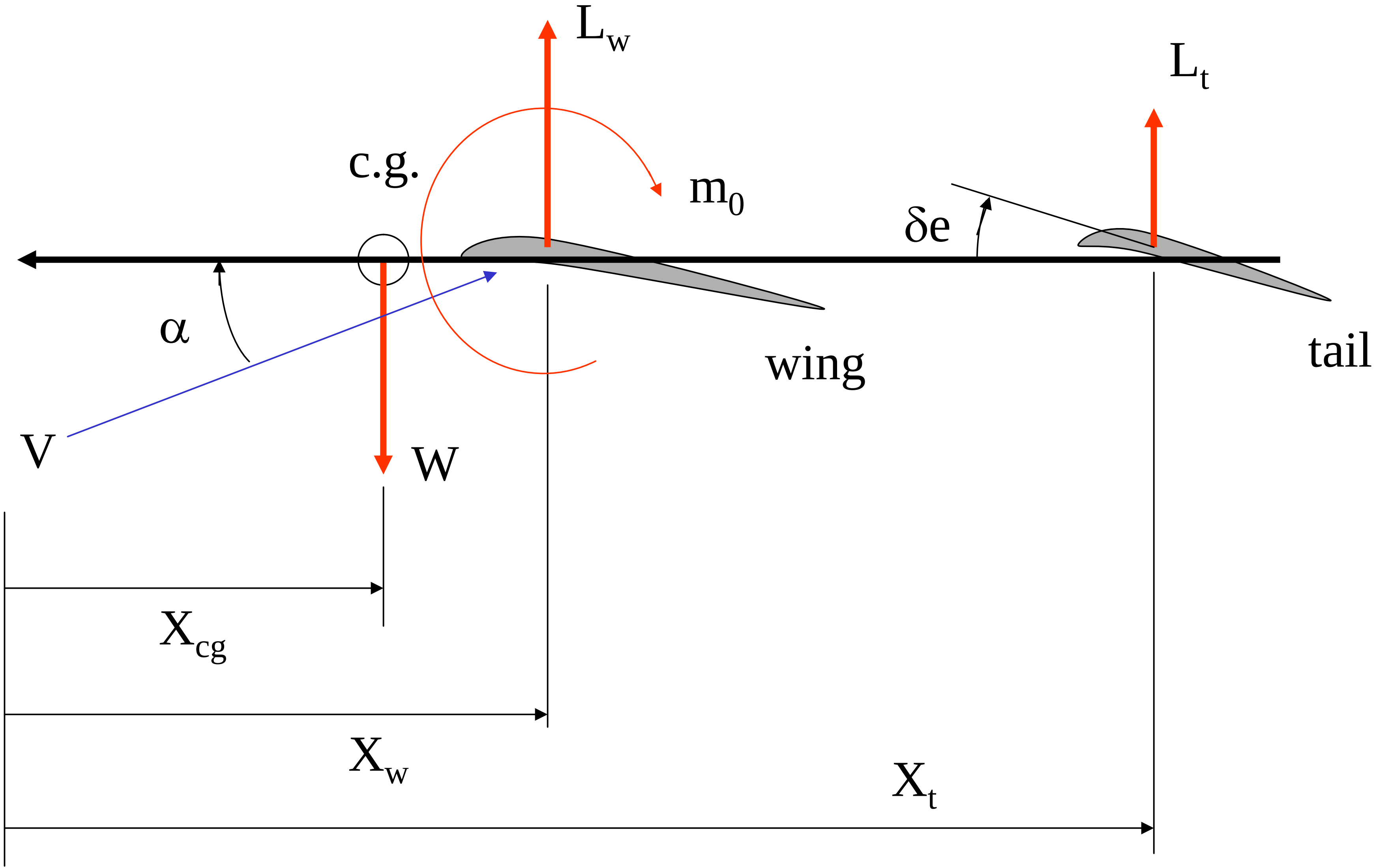
Moral

Aircraft stability depends critically on cg location

Note : All aircraft have forward and aft cg limits

Question : What happens if either forward or aft limits are violated ?

2.0 Quantifying static longitudinal stability



The *trim conditions* require that

$$L_w + L_t = W$$

$$m = 0$$

Where
$$m = m_0 + L_w (X_w - X_{cg}) + L_t (X_t - X_{cg})$$
$$= m_0 + L_w l_w + L_t l_t$$

2.1 : Lift curve slope

The key is to express aircraft lift and pitching moment in terms of contributions from the wing and the tail.

$$L_w = \tfrac{1}{2} \rho V^2 S_w CL_w$$

$$= \tfrac{1}{2} \rho V^2 S_W \; a_w \, \alpha$$

2.2 : Effect of downwash on the tail lift

Recall the tail surface is affected by downwash

$$L_t \quad = \quad \tfrac{1}{2} \rho V_t^{\,2} S_t CL_t$$

$$\quad = \quad \tfrac{1}{2} \rho (\eta V^2) S_t \; a_t [(1 - \partial\varepsilon/\partial\alpha) \alpha + \tau \delta e]$$

2.3 : Aircraft lift coefficient

Define the aircraft lift as $L = L_w + L_t$. In non-dimensional form

$$\tfrac{1}{2} \rho V^2 S_w CL \quad = \quad \tfrac{1}{2} \rho V^2 S_w CL_w \quad + \quad \tfrac{1}{2} \rho V_t^2 S_t CL_t$$

$$CL \quad = \quad CL_w \quad + \quad (V_t /V)^2 (S_t/S_w) CL_t$$

$$= \quad CL_w \quad + \quad \eta (S_t/S_w) CL_t$$

The aircraft lift curve slope $a = \partial(CL)/\partial\alpha$ is given by :

$$a = a_w + \eta (S_t/S_w)\, a_t\, (1 - \varepsilon_\alpha)$$

2.4 : Aircraft pitching moment coefficient

Define the aircraft pitching moment as :

$$m = m_0 + L_w l_w + L_t l_t$$

Non-dimensionalise i.e. divide by ½ $\rho V^2 S_w c$

$$Cm = Cm_0 + (l_w/c)\, CL_w + \eta\, \underbrace{(S_t l_t)/(S_w c)}\, CL_t$$

In terms of aoa and tail deflection

$$Cm = Cm_0 + (l_w/c)\, a_w\alpha + \eta\, V_H\, a_t\, [(1-\varepsilon_\alpha)\alpha + \tau\, \delta e]$$

Collecting terms…

$$\boxed{\begin{aligned} Cm = Cm_0 &+ [\ (l_w/c)\, a_w + \eta\, V_H\, a_t\, (1-\varepsilon_\alpha)\ ]\ \alpha \\ &+ [\ \eta\, V_H\, a_t\, \tau\ ]\ \delta e \end{aligned}}$$

How would you interpret this ?

2.5 : Neutral point & static margin

Differentiate Cm with respect to α

$$Cm_\alpha \quad = \quad (l_w/c)\, a_w + \eta\, V_H\, a_t\, (1- \varepsilon_\alpha)$$

Express in terms of cg location…

$$Cm_\alpha \quad = \quad [(X_w - X_{cg})/c]\, a_w + \eta\, (S_t/S_w)\, [(X_t - X_{cg})\, /c]\, a_t\, (1- \varepsilon_\alpha)$$

$$= \quad [\, (X_w/c)\, a_w + \eta\, (S_t/S_w)\, (X_t/c)\, a_t\, (1- \varepsilon_\alpha)\,]$$

$$-\ [\, a_w + \eta\, (S_t/S_w)\, a_t\, (1- \varepsilon_\alpha)\,]\, (X_{cg}/c)$$

$$Cm_{\alpha} \quad = \quad [(X_w - X_t)/c]\, a_w \quad + \quad a\,(X_t/c) \; - \; a\,(X_{cg}/c)$$

The neutral point is the cg location where $Cm_{\alpha} = 0$, i.e.

$$X_{np}/c \quad = \quad [(X_w - X_t)/c]\,(a_w/a) \quad + \quad (X_t/c)$$

Rewrite the pitching moment curve slope in terms of X_{np}/c …

$$Cm_{\alpha} \quad = \quad a\,(X_{np}/c) \; - \; a\,(X_{cg}/c)$$

$$= \quad -\, a\, \underbrace{[\, (\, X_{cg} - X_{np})\, /c \;]}_{}$$