Lecture 2 : The Flight Environment

*Or deriving atmospheric models*
1.0 The Earth’s atmosphere

Question: What is the Earth’s atmosphere?
**Question:** Are the boundaries for the atmospheric regions the same everywhere on the Earth?

No. The Earth is not a perfect sphere.

The World Geodetic System (WGS) models the Earth as an oblate spheroid.

- **Equatorial axis**  =  6,378,137.000 m
- **Polar axis** = 6,356,752.314 m
- **Polar tropopause** = 6 km
- **Equatorial tropopause** = 17 km
**Question**: Is the Earth’s atmosphere uniform?

<table>
<thead>
<tr>
<th></th>
<th>0 km</th>
<th>20 km</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air pressure (N/m²)</td>
<td>101 325</td>
<td>6000</td>
</tr>
<tr>
<td>Air density (kg/m³)</td>
<td>1.225</td>
<td>0.1</td>
</tr>
<tr>
<td>Air temperature (°C)</td>
<td>30</td>
<td>-60</td>
</tr>
</tbody>
</table>

**Question**: Any implications for aircraft?
Air temperature falls at a constant rate in the troposphere.

From the tropopause, the temperature remains constant at -60 °C until ≈20 km above S.L.

The lower stratosphere is the limit for atmospheric flight.
2.0 Aerodynamic flow parameters

Airspeeds can range from $10^0 - 10^3$ m/s

For this range of speeds, airflow characteristics are determined by 2 important parameters:

1. Reynolds number $Re$

2. Mach number $M$
2.1 Reynolds number

1. Air is “sticky” or viscous

2. From the aircraft’s viewpoint, the air at the surface is stationary

\[ V = \text{airspeed} \]

- velocity builds up from 0 to \( V \)
- normal direction
- aircraft surface
3. The thin region where the air flow builds up its speed is called the *boundary layer*.

4. The Reynolds number is a measure of the importance of this viscous effect

\[
Re = \frac{\text{(pressure forces)}}{\text{(viscous forces)}} = \frac{(\rho V^2)}{(\mu V/L)} = \frac{VL}{\nu}
\]

$L$ : reference length  
$\mu$ : coefficient of viscosity  
$\nu = \mu/\rho$ : kinematic viscosity
Example: What are typical Reynolds numbers for aircraft?

Using the F16 at S.L.

\[ V : 360 \text{ m/s (cruise)} \quad \quad \quad \quad \quad \quad L : 9.14 \text{ m (wing span)} \]
\[ \nu : 1.4607 \times 10^{-5} \text{ m}^2/\text{s} \text{ (kinematic viscosity for air at S.L.)} \]

\[ \text{Re} = \frac{V L}{\nu} \]

\[ = \]

\[ = \]

\[ \text{Re} \] is typically in the region of \(10^7 - 10^8\) for aircraft flight.

The lower the \(\text{Re}\), the stronger the viscous effect.

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2.2 The Mach number

1. Air is compressible.
2. A moving aircraft disturbs the surrounding air.
3. These disturbances, e.g., pressure variations, take a finite time to propagate at the speed of sound through the surrounding air.
4. The Mach number measures the importance of this compressibility effect.

\[ M = \frac{\text{airspeed}}{(\text{speed of sound})} = \frac{V}{a} \]
Example: What are typical Mach numbers for aircraft?

Using the F16 at S.L.

\[
V = 360 \text{ m/s (cruise)} \\
a = 330 \text{ m/s (speed of sound at S.L.)}
\]

\[
M = \frac{V}{a}
\]

\[
= \frac{360}{330}
\]

\[
= 1.09
\]

\[
M \text{ is typically in the region of } 0.8 - 2.0 \text{ for aircraft flight.}
\]

The higher the \( M \), the stronger the compressibility effect.
Classification of flow regimes via speed

- \( M < 0.8 \)  subsonic  incompressible aerodynamics
- \( 0.8 < M < 1.2 \)  transonic  localized compressibility effects
- \( 1.2 < M < 5 \)  supersonic  compressible aerodynamics
- \( M > 5 \)  hypersonic  aerodynamic heating
3.0 Atmospheric models

3.1 The equation of state

1. Pressure, density and temperature of air are related via an equation of state

2. The equation of state for an *ideal gas* is:

\[
P = \rho R T
\]
3.2 The variation of the speed of sound

1. The speed of sound varies with temperature as:

\[ a = (\gamma R T)^{1/2} \]
Ex: Find the temperature range within the troposphere, i.e. $h = 0$ to $11$ km

**Hint**: To convert from Centigrade to Kelvin

$$T \ (\text{K}) = 273.15 \ + \ t \ (^0\text{C})$$

<table>
<thead>
<tr>
<th>Altitude (km)</th>
<th>Temperature ($^0\text{C}$)</th>
<th>a (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>-55</td>
<td></td>
</tr>
</tbody>
</table>
Exercise : Show that

i) \( a = 20.06 \sqrt{T} \)

ii) \( a = (\gamma \frac{P}{\rho})^{1/2} \)

\[
\begin{align*}
a &= (\gamma R T)^{1/2} \\
&= \\
&= \\
&= \\
&= \\
\end{align*}
\]

ii) Hint : From the equation of state, \( P = \rho RT \)
3.3 The standard atmospheric model
or how pressure, density and temperature varies with altitude

1. Recall that in the troposphere, the temperature decreases at a constant rate with altitude i.e. \( \frac{dT}{dh} = -c = -6.51^\circ C/km \)

Hence for the troposphere \( T = T_{SL} - c \cdot h \), or

\[
\frac{T}{T_{SL}} = 1 - \left(\frac{c}{T_{SL}}\right) h
\]
2. Consider a small layer of air $dh$ above an area $A$. The decrease in pressure at the top is:

\[
dP = \frac{\rho g dA}{dh}
\]

Hence pressure and density are related by the aerostatic equation

\[
dP/dh = \frac{\rho g}{\rho}
\]
3. From the equation of state, \( \rho = \frac{P}{RT} \), hence

\[
\frac{dP}{P} = \frac{-(g/R)}{T_{SL} - c \ h} \ dh
\]

Integrating from sea level \( (h = 0) \), to desired altitude

\[
\frac{P}{P_{SL}} = \left[ 1 - \left( \frac{c}{T_{SL}} \right) h \right] \frac{g}{(Rc)}
\]

\[
= \left( \frac{T}{T_{SL}} \right)^{5.2506}
\]

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The density variation in the troposphere is obtained as:

\[
\frac{\rho}{\rho_{\text{SL}}} = \frac{P/P_{\text{SL}}}{T/T_{\text{SL}}} = (T/T_{\text{SL}})^{4.2506}
\]

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4. For the lower stratosphere, temperature is a constant \( T_{ST} \). Hence the aerostatic equation simplifies to

\[
\frac{dP}{P} = -\frac{g}{(RT_{ST})} \, dh
\]

Integrating from the tropopause \( (h = h_{ST}) \) to the desired altitude:

\[
\frac{P}{P_{ST}} = \exp\left[-\frac{g}{(RT_{ST})} \, (h - h_{ST})\right]
\]
The density variation with altitude in the lower stratosphere is derived simply by:

\[
\frac{\rho}{\rho_{ST}} = \frac{P}{P_{ST}} = \exp\left[ -\frac{g}{RT_{ST}} (h - h_{ST}) \right]
\]

Why?
Homework

1. Derive a table of temperature, pressure, density for $0 \leq h \leq 20$ km (at intervals of 200 m) for the equatorial atmosphere i.e. tropopause at 17 km.

2. Compare your table for $T_{SL} = 30^\circ C$ with international standard atmosphere (ISA) tables (sea level temperature = $15^\circ C$, tropopause at 11 km).