Lecture 4: Static Longitudinal Control

Or generating the correct control forces

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1.0 Types of flight control systems

Provides pilot with a natural "feel" for required control forces.

2. *Irreversible* : power assisted control - control surfaces actuated by hydraulics or electrical means e.g. in fly-by-wire systems (F16).
Required when control forces exceed human capabilities.
1.2 Human control force limits

FAR part 23 or 25, section 143, paragraph (c)...

... In no case may the control forces under the conditions specified in paragraphs (a) and (b) of this section exceed those prescribed in the following table:

<table>
<thead>
<tr>
<th>Values in pounds force applied to the relevant control</th>
<th>Pitch</th>
<th>Roll</th>
<th>Yaw</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) For temporary application:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stick</td>
<td>60</td>
<td>30</td>
<td>.....</td>
</tr>
<tr>
<td>Wheel (Two hands on rim)</td>
<td>75</td>
<td>50</td>
<td>.....</td>
</tr>
<tr>
<td>Wheel (One hand on rim)</td>
<td>50</td>
<td>25</td>
<td>.....</td>
</tr>
<tr>
<td>Rudder Pedal</td>
<td>.....</td>
<td>.....</td>
<td>150</td>
</tr>
<tr>
<td>(b) For prolonged application</td>
<td>10</td>
<td>5</td>
<td>20</td>
</tr>
</tbody>
</table>
2.0 Control surface hinge moment

The control surface hinge moment refers to the moment needed to rotate the surface.

Consider an all moving tail.

Taking moments about the hinge point,

The hinge moment

\[ H = \]
Non-dimensionalize $H$ with the tail area $S_t$ and the tail mean aerodynamic chord $c_t$, and a reference airspeed $V$

The *hinge moment coefficient*

$$C_H = \frac{H}{(1/2 \rho V^2 S_t c_t)}$$

This may be expressed in terms of the tail AOA and tail deflection as:

$$C_H = C_{H_{applied}} + C_{H_{mass}} + \left( \frac{\partial C_H}{\partial \alpha_t} \right) \alpha_t + \left( \frac{\partial C_H}{\partial (\delta e)} \right) \delta e$$
2.1: Stick-fixed and stick-free controls

For *stick-fixed* control, the applied hinge moment is non-zero and is generated by the control system e.g. hydraulics, servo to hold the desired control deflection.

For *stick-free* control, the control surface is allowed to rotate freely. Hence the applied hinge moment is zero and the control deflection is *a function of the AOA*.

NB: Stick free control is applicable only to reversible control systems.
2.2 : Stick-free elevator deflection and stick-free static stability

Stick-free

Equilibrium

⇒ $\delta e_{\text{stick free}} = -\left[ C_{H\text{mass}} + \left( \frac{\partial C_H}{\partial \alpha_t} \right) \alpha_t \right] / \left( \frac{\partial C_H}{\partial (\delta e)} \right)$

This affects the stability of the aircraft. Why?
2.3 Setting the desired stick-free control deflection

Trim tab - a small movable surface, adjusted to generate enough aerodynamic moment to hold the elevator at some desired deflection

Note that the trim tab deflection $\delta t$ is set opposite to the desired control deflection
With the trim tab, the hinge moment is of the form

\[ C_H = C_{\text{Happlied}} + C_{\text{Hmass}} + (\frac{\partial C_H}{\partial \alpha_t}) \alpha_t + (\frac{\partial C_H}{\partial \delta e}) \delta e + (\frac{\partial C_H}{\partial \delta t}) \delta t \]

Assume the trim tab does not affect lift significantly so the trim \( \alpha \) and \( \delta e \) varies with airspeed \( V \) as follows:

\[ \alpha = A + B/V^2 \]
\[ \delta e = C + D/V^2 \]

Homework: Prove this
The trim tab is set for stick-free trim at one specified trim airspeed e.g. \( V = V_{TR} \)

stick-free \( \Rightarrow \) equilibrium \( \Rightarrow \)

Hence the required trim tab deflection is

\[
- b_3 \delta t = b_0 + b_1 (1 - \varepsilon_\alpha) (A + B/V_{TR}^2) + b_2 (C + D/V_{TR}^2)
\]

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The hinge moment as a function of trim airspeed is then

\[ C_H = C_{\text{Happlied}} + b_1 (1 - \varepsilon_\alpha)B \left( \frac{1}{V^2} - \frac{1}{V_{TR}^2} \right) + b_2 D \left( \frac{1}{V^2} - \frac{1}{V_{TR}^2} \right) \]

\[ = C_{\text{Happlied}} + \left[ b_1 (1 - \varepsilon_\alpha)B + b_2 D \right] / V^2 \left[ 1 - \left( \frac{V}{V_{TR}} \right)^2 \right] \]

For equilibrium \( C_H = 0 \), hence

\[ C_{\text{Happlied}} = - \left[ b_1 (1 - \varepsilon_\alpha)B + b_2 D \right] / V^2 \left[ 1 - \left( \frac{V}{V_{TR}} \right)^2 \right] \]
3.0 Stick forces

The stick force $P$ is usually mechanized to be proportional to the applied hinge moment

\[
P = G \cdot H
\]

\[
= G \left( \frac{1}{2} \rho V^2 S_t c_t \right) C_{H,\text{applied}}
\]
3.1 FAR stick force requirements

FAR 23.173 paragraph (a) A pull must be required to obtain and maintain speeds below the specified trim speed and a push required to obtain and maintain speeds above the specified trim speed.

i.e. pull (+ve) – increase aoa

push (-ve) – decrease aoa

Question : How is this realized for a tail controlled aircraft?
Example: Stick-elevator mechanization for tail controlled aircraft
3.2 FAR Stick force gradient requirements

FAR 23.173 (c) The stick force must vary with speed so that any substantial speed change results in a stick force clearly perceptible to the pilot.

FAR 25.173 (c) The average gradient of the stable slope of the stick force versus speed curve may not be less than 1 pound for each 6 knots.

\[ | \frac{\partial P}{\partial V} | > \frac{1}{6} \text{ lbf/knot} \]
The stick force gradient $\partial P/\partial V$ may be obtained as follows:

Stick force \( P \) = \[ G \left( \frac{1}{2} \rho V^2 S_t c_t \right) C_{\text{Happlied}} \]

\[ = - G \frac{1}{2} \rho S_t c_t [ b_1 (1 - \varepsilon \alpha) B + b_2 D] [1 - (V/V_{TR})^2] \]

Question: How does the stick force varies with aircraft size?

Differentiate the stick force \( P \) wrt \( V \)

\[ \frac{\partial P}{\partial V} = G \rho S_t c_t [ b_1 (1 - \varepsilon \alpha) B + b_2 D] \frac{V}{(V_{TR})^2} \]

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3.3 Variation of stick force with trim airspeed